EDITOR'S PREFACE

It is a pleasure to have a part in making this treatise on the Lanchester theory of combat available to the military operations research community, and to others who may be interested in the theory of combat. It was Lanchester's particular contribution to recognize that the same differential equations that can be used to explore the speed and extent of the reactions to be expected when two chemicals are mixed together could be used with little modification to describe the casualties one must expect when two opposing forces engage in a battle. The present book, not to mention its extensive bibliography, is witness to the fecundity of Lanchester's insight as a stimulus to combat analysis. The bibliography is, of course, a natural way to pursue this subject further. For people in the United States, another way is to join the Military Applications Section (MAS) of the Operations Research Society of America. For U.S. citizens with appropriate clearance, there is also the Military Operations Research Society. Another organization of particular interest is the United Kingdom Study Group on Lanchester Theory. This group has recently compiled an extensive bibliography of open-literature papers on Lanchester theory (available in the U.S. through MAS). It is also collecting "milestone papers" on Lanchester theory—the collection to be published in 1984. Its current address is c/o Dr. Fergus Daly, Faculty of Mathematics, the Open University, Walton Hall, MILTON KEYNES MK7 6AA, England.

By its very nature, Lanchester theory pertains primarily to what might be called continuous combat—combat in which the number of units on each side is large enough so that the loss of a single unit is not a discontinuity. This limitation, plus the many complications that
any battle can be cursed with, have spawned scenarios that have called for other approaches—particularly simulation models. These continue to provide much valuable insight into combat behavior. However, the entire analysis community (including the simulators) continually searches for more analytic strength. In the present context, this translates into three disiderata: (1) continue efforts to simplify combat descriptions so that existing Lanchester analysis can be exploited; (2) expand the family of Lanchester formulations; and (3) search for other powerful analytic techniques for the discontinuous cases.

Publication of analyses such as this in the "open literature" is an issue that has been given considerable thought, considering the quintessentially adversarial nature of combat. Why should the military analysts of different nations, particularly including potential adversaries, wish to co-operate in developing such a subject? In certain applications, it is clear that in fact they should not. Probably the scientific instinct of wanting to share one's results has occasionally led to inappropriate publications in combat analysis. One defensive "justification" is that, although the open availability of such studies may benefit one's adversary, the convenience within one's own country more than makes up for this drawback. This book (and others in this series) is published at least partly with more positive motivations. Contrary to popular conceptions, even military analysts recognize that combat is typically undesirable and should be avoided if at all possible. Analytic predictions of combat results can help make it clear to both sides just how undesirable various battles may be. A similar consideration
may be the need for "both sides" to develop better communications or understanding on analytical (as opposed to political) levels. International issues such as arms control, deterrence, and armistice negotiations would seem to be outstanding examples where we are already being forced to share analytic frameworks.

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PREFACE

The Twentieth Century has been characterized by innumerable attempts to use the Scientific Method as a basis for policy planning in national and international affairs. The emergence of the field of operations research (OR) out of attempts of scientists in the Western Democracies to apply the Scientific Method to military problems during World War II is well known. Since World War II there has been a dramatic growth in both the interest in and use of OR and systems-analyses techniques for such purposes within the U.S. defense establishment, especially since the beginning of the so-called McNamara Era of defense planning. A concomitant trend has been an equally dramatic increase in both the number and variety of mathematical models used to support these analytical activities.

Unfortunately, professional communications within the defense analytical community have not kept pace with this dramatic growth in modelling and analysis activities. In particular, there has been a relative lack of scientific communication and organization of knowledge concerning the foundations of defense analyses and associated defense-analysis technology. However, even this important point has not been explicitly articulated in several fairly recent critical appraisals of the foundations of defense analyses†. To be sure, research progress on these foundations has been made, but it has not always been efficiently and effectively communicated to interested parties. This inaccessibility

of scientific information concerning combat-modelling methodologies has contributed to the existing gap between theory and practice. Some undesirable consequences of this communications deficiency between analysts and researchers include (1) duplication of effort, (2) models being inefficiently used (or even misused), (3) lack of the appropriate intellectual environment for effective professional review by peers, and (4) lack of any "road map" to provide direction (and purpose) for methodological developments.

Thus, although there has been a great need, information about combat-modelling methodologies, their strengths and weaknesses, limitations, etc. has not been very widely disseminated in accessible form. National security (i.e. material being classified) has not really been a factor in producing this situation in which the quantitative foundations of defense analyses have not been readily available to the analysis community for scientific scrutiny. Without such generally available methodological material, little scientific progress can be made, since open scientific discussion is hampered by such vital information not being readily available to all interested parties. Consequently, this monograph has been written in an attempt to fill some of this void by organizing the current state of knowledge about a certain type of combat model, so-called LANCHESTER-type equations of warfare. Hopefully, its appearance will also stimulate discussion and debate concerning assessment of existing capabilities and future needs in this one specific area of combat-modelling methodology.

At the personal level, the reader may be interested in knowing how the author has become drawn to this subject: the author has been interested in the subject of LANCHESTER-type combat models since the late
1960's, when R. NICHOLS HAZELWOOD introduced him to combat models and, in particular, to the work of HERBERT K. WEISS. He has been fortunate enough to have subsequently had such interests nurtured at the Naval Postgraduate School (NPS) and has had the opportunity to do research on combat models and teach graduate-level courses about them to students (primarily U.S. Army and U.S. Marine Corps officers) in the OR curriculum at NPS since 1970. The treatise at hand (and its petite predecessor Force-on-Force Attrition Modelling††) has evolved from these activities.

This monograph is a comprehensive treatise on LANCHESTER-type models of warfare, i.e. differential-equation models of attrition in force-on-force combat operations. Its goal is to provide both an introduction to and current-state-of-the-art overview of LANCHESTER-type models of warfare as well as a comprehensive and unified in-depth treatment of them. Both deterministic as well as stochastic models are considered. Such models have been widely used in the United States and elsewhere for the modelling of force-on-force attrition over the complete spectrum of combat operations, from combat between platoon-sized units through theater-level air-ground combat. This material should be of interest primarily to individuals concerned with defense planning, quantitative aspects of military analysis, military OR, war gaming, or combat modelling, although it may also be of interest to the reader concerned with the modelling and analysis of other dynamic systems. It should also be of interest to the concerned citizen who is interested in the foundations for defense analysis and has the appropriate technical background.

†† The full citation here is JAMES G. TAYLOR, Force-on-Force Attrition Modelling, Military Applications Section of the Operations Research Society of America, Arlington, Virginia, 1980.
I have tried to make this monograph particularly suitable for three specific groups of readers: (1) the beginning student of military OR, (2) the practicing military OR analyst, and (3) the research worker in OR, applied mathematics, models, or systems analysis and evaluation. For the first group (i.e. beginning students of military OR), I have included much expository and explanatory material: each major topic is preceded by a general discussion of the contextual setting in which it arises (with figures depicting important conceptual ideas and typical numerical results). For these readers I have supplied motivation and overview. For the second group (i.e. practicing military OR analysts), I have emphasized those theoretical and applied concepts that are basic for the building and running of operational combat models (e.g. the numerical determination of values for LANCHESTER attrition-rate coefficients) and have provided a bridge between such current operational combat models and the abstract notions that form their conceptual bases. For these readers I have supplied examples from current operational combat models. For the third group (i.e. OR and other researchers), I have surveyed the current state of the art of pertinent quantitative methodologies concerning LANCHESTER-type combat models, particularly mathematical results for analytically investigating the quantitative behavior of relatively simple LANCHESTER-type models. For these readers I have included numerous references to the literature and a comprehensive bibliography on the LANCHESTER theory of combat. This book, however, is particularly slanted toward the beginning military-OR student who is interested in force-on-force combat models, since it is through him (particularly if he is an officer in one of the military services) and his education about combat models that the greatest long-term improvements in defense decision
making may be achieved by the U.S. Department of Defense (DoD). It strives to give the reader (regardless of his orientation) an appreciation of the complex operational models that are today used for investigating large-scale simulated air-ground combat operations by DoD. Mathematical prerequisites have been kept to a minimum, with more mathematically oriented sections that are not necessary for the understanding of the sequel being identified as "starred sections." Throughout this monograph, modelling aspects have been emphasized. Anyone with a background in calculus good enough to understand the physical interpretation of an ordinary-differential equation model should have no trouble in reading most of it. However, the few starred sections do require more mathematical sophistication to be understood.

This monograph is organized into two volumes of four chapters each. The monograph begins with a discussion in Chapter 1 about the general nature of models (particularly, combat models), their use in OR, and particularly the contextual setting for the use of such models as planning tools in the U.S. DoD. Chapter 2, which begins by reviewing FREDERICK W. LANCHESTER's pioneering work on quantitatively justifying the Principle of Concentration, examines LANCHESTER's classic combat models and the many subsequent variants of them. The models are kept simple and deterministic here, but the stage is set for subsequent model enrichments considered later in this monograph. The discussion of LANCHESTER's classic combat models is self-contained, with background material on the relevant mathematics being contained in an appendix. This material is fundamental and very important not only in its own right but also for understanding subsequent developments in this book: it forms the basis for the many extensions considered later in the book. A
selection of problems has been provided in Chapter 2 for the enhancement of the reader's familiarity with these basic models.

Chapter 3 contains a comprehensive examination of some simple models of battle termination. It considers both the empirical foundations of such models and also the mathematical analysis of their properties. Both deterministic and stochastic battle-termination processes are examined, although only deterministic LANCHESTER-type attrition processes are considered. This chapter is essentially a state-of-the-art survey of battle-termination modelling and focuses on work by H.K. WEISS and R.L. HELMBOLD. It culminates by examining HELMBOLD's empirical investigation of the validity of breakpoint hypotheses. Chapter 4 examines stochastic versions of the simple deterministic homogeneous-force models considered in Chapter 2. Continuous-time MARKOV-chain models of LANCHESTER-type attrition processes are exclusively considered. After examining analytical results for such models and noting their complexity, the reader will certainly appreciate the fact that except for small numbers of combatants, the expected course of combat (at least for MARKOV-chain models of homogeneous-force combat) is well approximated by deterministic LANCHESTER-type equations. Not surprisingly, such deterministic LANCHESTER-type models are consequently frequently referred to as expected-value models. Herein ends Volume I.

Volume II begins with Chapter 5. In order to use a LANCHESTER-type model in any actual military OR study, numerical values must be determined for the attrition-rate coefficients, which represent the single weapon-system-type kill rates. Chapter 5 considers in detail approaches and methodologies for determining such numerical values for LANCHESTER attrition-rate coefficients for various types of weapon systems. The
two main approaches that are currently used in the United States to
determine such single-system kill rates are based on using (1) a "free-
standing" analytical submodel of an individual firer engaging a single
enemy target, and (2) a statistical estimate based on "combat" data
generated by a detailed Monte Carlo combat simulation. Such methodology
is a basic essential ingredient for the building of any operational
LANCHESTER-type combat model. Chapter 6 considers LANCHESTER-type
models for combat between two homogeneous forces and emphasizes the
analysis of such models. For several important classes of homogeneous-
force models, analytical results are given that make the analysis
(including determining the force levels as functions of time and predict-
ing the battle's outcome) of such variable-coefficient combat models
almost as convenient as that of LANCHESTER's original constant-coefficient
ones. Tables of special new mathematical functions (i.e. the LCS
functions developed by the author) are provided for the reader's use in
analyzing certain important classes of "aimed-fire" battles between two
homogeneous forces.

Chapter 7 considers modelling tactical engagements and surveys
approaches currently used in the United States for assessing casualties
in simulated tactical engagements between general-purpose military
forces in conventional air-ground combat operations. It reviews the
various different modelling alternatives available to the military OR
worker and then expounds on both detailed deterministic LANCHESTER-type
models of attrition in tactical engagements and also aggregated-force
models based on index numbers (e.g. firepower scores), with hierarchical
modelling approaches also being briefly discussed. Model formulation
and methodological aspects are emphasized, with simple auxiliary models

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being used to illustrate modelling points for developing and understanding complex operational models. Examples of current operational models that use the two main theoretical approaches of casualty assessment (i.e. detailed LANCHESTER-type force-change representations and aggregated-force casualty assessments based on index numbers) are given. Recent developments by authors such as L.B. ANDERSON, D.P. DARE, and R.M. THRALL for determining firepower scores (i.e. weapon-system-type values) from a linear model that imputes values to weapon-system types based on their LANCHESTER attrition-rate coefficients are reviewed and discussed, as well as the important (and elusive) problem of historical validation of attrition models. Next, Chapter 8 reviews work on developing insights into the structure of optimal tactical decisions by applying the appropriate optimization theory to a combat model with military strategy and tactics quantified through tactical-choice variables. Gaming aspects are also briefly considered. This chapter is essentially a comprehensive overview and review of work on the quantitative study of military strategy and tactics by using optimization theory in conjunction with combat-modelling theory. Again, simple auxiliary LANCHESTER-type models are used to study these complex operational problems. As before, model formulation and insights gained into the structure of optimal time-sequential decisions are stressed, with optimization-theory (i.e. differential-game) prerequisites being kept at a minimum (i.e. the results of such optimization studies are given but not the details in the application of the optimization theory). Finally, a comprehensive bibliography on the LANCHESTER theory of combat is included in an appendix for the reader who is interested in further information about it.

This monograph has evolved out of a tutorial on LANCHESTER-type
models of warfare that the author was invited to deliver by the Military Applications Section of the Operations Research Society of America (ORSA) at the 46th National ORSA Meeting on Thursday October 17, 1974 in San Juan, Puerto Rico. This tutorial was well received, and it was subsequently repeated at the 35th Military Operations Research Symposium in July 1975 and at the 15th Annual U.S. Army Operations Research Symposium in October 1976. After attending this tutorial in July 1975, CDR JAMES J. MARTIN, USN, then Chairman of the MORS Publications Committee, expressed strong interest in the author's expanding the tutorial material into a monograph on LANCHESTER-type models of warfare. The writing of this monograph was consequently begun under the sponsorship of the Office of Naval Research (Code 431, Naval Analysis Programs) in July 1976. Continued encouragement by Dr. MARTIN (now retired from the U.S. Navy) has been appreciated. I have used earlier drafts of the beginning portions of this material (primarily Chapters 1 and 2 and occasionally Chapter 3) in graduate courses on combat models for OR students at the Naval Postgraduate School.

The author would like to thank all the organizations and individuals who have helped facilitate the appearance of this monograph. Although all those who have helped me are far too numerous to mention, I would like to explicitly express my thanks to several. In particular, the writing of this monograph has been financially supported by the Office of Naval Research (both through direct funding by Code 431 and also through the Foundation Research Program at the Naval Postgraduate School), the U.S. Army Research Office (ARO), Durham, North Carolina, and the Headquarters of the USAF, Studies and Analysis Group. Additionally, ARO supported some separate research during this period on
LANCHESTER-type models of warfare, and results from this work have been incorporated into the monograph at hand. Most of the author's research on LANCHESTER-type models of warfare, however, has been supported over a number of years by the Office of Naval Research (both through direct funding by Code 431 and also through the Foundation Research Program at NPS). The author would like to thank Provost JACK R. BORSTING of NPS (formerly chairman of the OR department) for his continual encouragement and support of such work as well as that from subsequent OR department chairmen Dean DAVID A. SCHRADY and Professor MICHAEL G. SOVEREIGN. The endeavors of Associate Professor GILBERT T. HOWARD (associate chairman for research of the OR department) in this respect are also gratefully acknowledged. The author would also like to thank HERBERT K. WEISS, Dr. JAMES J. MARTIN, Dr. FRANK E. GRUBBS, Professor MARTIN SHUBIK, and LTC JOHN FRIEL (USAF), for their constant encouragement. Additionally, the author would like to thank Professors CLINTON J. ANCKER, GORDON E. LATTA, GUILLERMO OWEN, and MICHAEL G. SOVEREIGN, as well as LTC RICHARD S. MILLER (USA) for their numerous suggestions for improving this manuscript. I am especially indebted to LTC MILLER for many stimulating discussions on the topics of combat modelling and this constant encouragement and help concerning this project. The author would also like to thank the late ROSEMARIE STAMPFEL for her consummate typing of this manuscript. Finally, the author would like to thank his family for their understanding of the long hours he has spent

††† Sadly and unexpectedly ROSEMARIE STAMPFEL passed away just after completing the typing of the first draft of the manuscript. As a technical typist, she was without peer. I would like to thank her for her many suggestions and help in improving this manuscript. She will be missed by many.
writing this book and for their constant support, especially his wife MARY ANN, who has proofread most of this monograph (some while recovering from surgery).
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Chapter 1. BACKGROUND AND INTRODUCTION


Loosely speaking, LANCHESTER-type models of warfare are differential-equation models of combat operations. In one form or another, such models are fairly widely used in operations research (OR) studies by the Department of Defense (DoD) in the United States. The use of these combat models for planning purposes has been made possible by modern large-scale digital-computer technology. However, there are competing methodologies (for example, so-called high-resolution Monte-Carlo simulation) for combat modeling, and there has been much debate\(^1\) by advocates about the advantages of this method or that one for defense planning. To place such discussion about the use (and misuse of combat models, their realm of applicability, and their strengths and weaknesses in proper perspective, it seems appropriate to briefly discuss the nature of OR, combat models, and their use by DoD. The reader should keep in mind, however, that this book will focus on LANCHESTER-type models of warfare.


Operations research (OR) originated out of questions arising in military activities during World War II. After the war, the approach and techniques of OR were applied to business and non-military government problems. OR has expanded greatly during the thirty or more years since the end of World War II. What exactly is OR? Although there is far from universal agreement\(^2\) as to the exact nature of OR, the author prefers to think of OR in the following terms\(^3\): operations research is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control.
The above definition of OR is not new, but the author feels that it is important because this definition focuses on what is being done and not the techniques used. Moreover, one should expect to find that different methodologies receive different amounts of emphasis in different fields of application of OR. For example, in the private (i.e. business) sector of the economy one finds that the "theory of the firm" and related subjects (such as profit maximization, efficient distribution of products, investment planning, inventory management, etc.) play a central role in OR applications and require the use of certain OR theory and techniques (such as inventory theory, queueing theory, linear and integer programming, discounted cash flow, etc.). One would expect quite a different phenomenological basis for defense planning, with possibly different OR techniques receiving emphasis. It is the author's hypothesis that defense planning should be based as much as possible on the scientific study of warfare. Unfortunately, this is not the case in practice today (see, for example, SHUBIK and BREWER [86, pp. 9-10] for a discussion of this point). For further discussion of the nature of OR, the interested reader should consult the literature.

Four concepts of fundamental importance to the practice of OR are (see HERRMANN and MAGEE [38]):

(C1) the model,
(C2) the measure of effectiveness (MOE),
(C3) decision making,
(C4) the role of experimentation.

Models (in particular, so-called LANCHESTER-type models of warfare) are the central theme of this book. We should bear in mind, however, that the development and application of a model in an OR study is only one of several essential ingredients for a successful study. Each of the three other aspects
listed above can significantly contribute to the failure of a defense-planning study. It is the author's opinion that people unfamiliar with quantitative models are quick to blame an unfamiliar modelling methodology for deficiencies in the application (e.g. data-base quality or errors, incorrect implementation, etc.) of a particular model. The practitioner should not blame the model (particularly, a LANCHESTER-type model) if the wrong MOE is used in a study, nor should he blame the modelling methodology if the model is incorrectly applied or exercised with low-quality data, or if the scenario is wrong. Thus, the development of a combat model is only one facet of a military OR study, albeit a very important aspect.

During World War II most OR concerned actual ongoing military operations. Some people prefer to use the term operations analysis (OA) for such activities. In 1976 (with the end of U.S. involvement in Southeast Asia) most applied military OR activities concerned some type of planning. If a military system does not physically exist (and even when it does), its effectiveness must be evaluated "on paper." Thus, for example, for assistance in system-acquisition decisions, one would expect to use in the advanced planning phase some type of combat model to help quantitatively explore the possible benefits from a proposed system. Even if a prototype has been built and "operational" data has been collected, some type of combat model may be required to assess the system's military worth based on the observed performance data. In other words, the nature of military OR has changed since World War II when few operational models were really used, and today combat models are an essential (and expensive) part of DoD planning activities.

1.1.2. The General Nature of Models.

It seems appropriate for us to briefly discuss the general nature of
models in order to better place combat models in proper perspective. Models are basically representations. They may be representations of states, objects, or events. Models are idealizations (i.e. abstractions) in the sense that they are less complicated than reality (and hence potentially easier to use for research purposes). The U.S. Army Models Review Committee [42, Appendix B to Chapter I] has defined a model as "an abstract representation of reality which is used for the purpose of prediction and to develop understanding about the real-world process."

Thus, models are easier to manipulate and "carry about" than the real thing. They are relatively simple compared with reality because only the relevant features of reality have been represented. For the person unacquainted with this basic property of models, however, it is easy to confuse relevance with realism. Thus, many DoD decision makers who are removed from the modelling business find simulations to be more credible models of combat operations than analytical models because of the much larger amount of detail that is present in a simulation. Additionally, models allow one to transcend one's environment and make inferences about things and events that have not been experienced directly. In the analysis of combat operations (particularly possible future ones), this aspect is quite important.

There are many ways to classify models. Three different basic types of models are the following:

(T1) iconic models,

(T2) analogue models,

(T3) symbolic models.

An iconic model is a large- or small-scale representation of states, objects, or events. They "look like" what they are supposed to represent with only
a transformation of scale. Examples of iconic models are a flow chart, blueprint, road-map (or any other type of picture or diagram that looks like the real thing), pilot plant, or a wind tunnel. In each case only the scale of the system or operation has been changed.

An analogue model uses one property to represent another different property. For example, we can represent the third dimension (i.e. elevation) on a two-dimensional map by means of contour lines, which represent information about changes in elevation (i.e. slopes) by their distance apart. Another similar example is the use of colors to represent different types of terrain on a map. Since one property is used to represent another, a legend is required to remind the reader of the transformation of properties. Other examples of analogue models are the slide rule and an electrical system represented by a hydraulic system.

The last general type of model is the symbolic model, which represents properties symbolically. Verbal descriptions of processes or systems qualify as symbolic models. When symbols represent quantities, the model is usually called a mathematical model. We will focus on mathematical models of combat (in particular, combat attrition) in this book. Here we have indicated to the reader, however, that other types of models certainly exist.

Although they are the most abstract, the distinguishing feature of mathematical models is the ease with which they may be manipulated for the extraction of information. Iconic and analogue models are much less flexible in this respect. In terms of combat operations, we should point out that field exercises are basically iconic models, while map exercises are basically analogue models. However, both these two types of combat models are difficult to manipulate (particularly the field exercise, which is also very costly). Thus, although they may require some time and cost to develop, mathematical models are relatively easy to manipulate and hence
respond to the demands of analysis.

Many other classifications of models are possible, but for our purpose of studying combat modelling we need only distinguish here between two basic types of mathematical models:

(T1) deterministic model.
and (T2) stochastic model

A deterministic model is one that contains no element of chance. Hence, its output is uniquely determined by its input in the sense that the same input always produces the same output. A stochastic model contains an element of chance (or uncertainty) so that its output is not uniquely determined in this sense by input, but rather one must talk about the chances of observing various outputs for a given input. In other words, one must consider the probability distribution over the set of possible outcomes for a given set of inputs. In this book we will consider both deterministic and stochastic LANCHESTER-type models of warfare.


The Twentieth Century has been characterized by attempts to use the Scientific Method in policymaking, in particular for military and defense problems. Many writers have stressed the importance of applying quantitative OR methodologies to defense planning. Enlightened defense planning is, of course, important for both the short-run and also the long-run national security of the United States. What are typical defense-planning problems? According to STOCKFISCH [90], they are as follows:

(P1) How do we assess a possible opponent's military capability, and how large should our military forces be to meet the perceived threat?
(P2) How should the total force be structured between major services, such as land forces and tactical air forces?

(P3) How should the land forces be structured with respect to (1) combat branches, such as infantry and tanks, and (2) service specialties that provide logistic and personnel support?

(P4) What should be the technical performance and physical specifications of new weapons that will be the object of engineering development programs? Given the availability of new weapons, what should be their tactical usage, how many of them should be procured, and in what organizational and command context should they be employed?

Such questions concern the evaluation of weapon-system and force-level planning alternatives in future time frames. In order to determine the benefits to be gained from a particular alternative, one is invariably faced with the problem of predicting the effectiveness of specified military forces in possible future military engagements. Since such forces and/or weapon systems only exist "on paper," some type of combat model (see Section 1.3 for further details) must be used in such studies. In way of summary, then, combat models are valuable in many aspects of defense planning: (1) for evaluating "on paper" proposed weapon systems during advanced planning; (2) for extending, interpolating, and interpreting operational test data during field testing; etc. (see [104] for a fuller discussion).

Thus, combat models have been used as decision aids for defense planning. They have actually been used by analysts to study such major subjects (see STOCKFISCH [90]) as:

(S1) the design specification and selection of new weapons,

(S2) the allocation of resources between air and land forces and, within land forces, between infantry and artillery,
(S3) how tactical air capability might be allocated among diverse missions,
(S4) the amount of logistic support that the combat elements of field forces should have,
(S5) the rate at which forces might be mobilized and deployed, and (S6) the issue of how large the forces should be.

The kinds of models that are used for such studies should be related to the type of information that is desired from the analysis. We will discuss the various types of combat models in the next section.

If one contrasts World War II operations research with today's practice, then it is clear that a major change has occurred in the practice of military OR and the use of models in defense planning. OR has ceased to be a purely scientific discipline, and some, in fact, feel that it has become a purely speculative activity (see, for example, BONDER [9]). During World War II, operations research was primarily concerned with the engineering (i.e. designing and planning) of on-going operations. Consequently, some combat data could be collected as needed for use in studies. Hypotheses about such military operations might actually be scientifically verified by testing against this data. Thus, World War II OR was many times a truly scientific discipline. Today military operations research is primarily concerned with planning of some type; and, as emphasized by BONDER [9], it has ceased to be a truly scientific discipline because of the absence of combat data (see also HOWLAND [46]).

In this vein, SETH BONDER [10] has emphasized that there are almost no empirically verified models of most combat processes. Besides the inherent problem of operational definition and measurement, the major insuperable difficulty in empirically verifying any combat model is that
the historical data base is too poor: it is not rich enough in detail to permit the classic scientific verification of combat models, since nations fight wars for other reasons than to collect combat data. Unfortunately, in the past military historians have been surprisingly reluctant to provide information on battles such as the number of forces of each kind participating on both sides and the losses. H. K. WEISS [115] feels that "the average military historian is particularly susceptible to the criticism aimed by VAGTS [102] (see also [103]) at the 'average military officer' of avoiding 'bellometrics' 'as something too materialistic and derogatory to military art.'"

This shortage of historical and other empirical data for combat models and analysis is apparently not as widely acknowledged, articulated, or appreciated by the policy-making community (and even some parts of the analysis community) as it should be (see also STOCKFISCH [90]). Moreover, one cannot expect accurate point estimates of combat effectiveness from these models. Rather, such nonempirically developed models should only be used for analysis purposes to provide defense management with [9]:

(R1) insights into directions and trends thereby increasing understanding of the system dynamics,

(R2) guidelines for the development of data-collection plans — what data is important and how accurate it must be,

(R3) guidelines for the development of technological and modelling research plans.

It is in this spirit of developing insights that simplified LANCHESTER-type models of warfare are considered in this book. In the same vein, KARL von CLAUSEWITZ\textsuperscript{13} [20, p. 191] stated many years ago in his classic work On War that if theory caused a more critical study of war, then it had achieved its purpose.
Underlying the engineering (i.e. designing and planning) of military operations, evaluation of military systems, and other problems of defense planning, however, should be the scientific study of conflict (in particular warfare). Just as most branches of engineering (for example, mechanical engineering) are based on NEWTONIAN physics, so should military operations research be based on the scientific study of warfare. Unfortunately, appallingly little basic research on conflict and warfare has apparently been conducted. No science of "bellometrics" [102; 115] has as yet emerged. Later in this book we will briefly discuss what has been done with respect to the scientific verification of LANCHESTER-type models of warfare. As mentioned above, the quality and extent of the historical data base have been severely limiting factors for such important investigations.

1.3. Different Types of Combat Models.

As we have discussed in Section 1.1.2. above, models are representations of reality, and we have seen that different types of such representations are possible. With respect to combat operations, Figure 1.1 shows the variety of forms that combat models may take. One can associate trends in model characteristics such as degree of operational realism, abstraction, and convenience and accessibility with this spectrum of combat models. As Figure 1.1 shows us, operational realism and degree of abstraction are conflicting qualities.

For present purposes, let us focus on the three right-most types of combat models depicted in Figure 1.1. Following BONDER [10], we will limit our discussion of combat models to the following three general types:

(T1) war games,
(T2) simulations,
(T3) analytical models.
Figure 1.1. The spectrum of types of combat models.
Additionally, in the ensuing discussion we will generally emphasize ground combat models (i.e. models of warfare between ground combat units). Although other classifications are certainly possible, the above is adequate for now.

According to PAXSON [70], "a war game is a model of military reality set up by a judicious process of selection and aggregation, yielding the results of the interactions of opponents with conflicting objectives as these results are developed under more or less definite rules enforced by a control or umpire group." The distinguishing feature of war games in relation to simulations and analytical models, however, is that actual human beings are used to simulate decision processes by having people play the roles of decision makers and use their own judgments in making decisions (see also [42]). This distinction is graphically depicted in Figure 1.2.

War games may be classified as being either "rigid" or "free", depending on whether or not the assessment rules are rigidly prescribed and completely cover all possibilities. These two types of war games (i.e. the rigid and free war games) correspond to the opposing demands of realistic games and playable games. The rigid war games are somewhat similar to simulations in their assessment of combat outcomes in that combat interactions are considered in detail. Before the age of large-scale computers, the sheer immensity of the volume of the details for such rigid assessments was overwhelming: it was not uncommon for many volumes (i.e. books) of rules and combat-results tables to be required for the running of a rigid war game. As a reaction and revulsion to such overwhelming detail, "free" war games were developed, with the assessment of combat outcomes being judgmentally determined by umpires. It is interesting to note that modelling issues such as degree of resolution,
Figure 1.2. Distinction between different types of combat models according to how decision making is represented.
appropriate technique of aggregation, amount of detail, etc. were all considered in the past by war gamers of the 19th and 20th centuries.

Today many computer-assisted war games exist, with the computer doing the bookkeeping and assessing combat outcomes. To a certain extent, the modern large-scale digital computer has neutralized some of the shortcomings of rigid war games. Teams of players typically represent the commanding officers and their staffs. However, this type of model, i.e. the rigid (computer-assisted) war game, is very expensive in terms of time and money to develop, maintain, and use. BONDER [10] points out that it typically may take something like four to eight years to develop such a rigid war game. He also notes [10, p. 73] that as recently as 1971 it took six months to obtain one realization of ten hours of battle with a particular war game. War games may be an excellent vehicle for developing general insights and identifying critical elements for further more detailed analysis, but many feel that this type of model is not a feasible vehicle for systematically analyzing a wide variety of system alternatives in a responsive manner [10].

To simulate means to act like. Simulations are models in which processes and activities are "acted out." Systems are microscopically analyzed and modelled by analogue duplication. Because of the large amount of bookkeeping involved in such minute duplication, a large-scale digital computer is a necessity. In fact, the development of the modern digital computer has led to the widespread use of simulation as an analysis technique. Such simulation of combat operations is the modern-day automated version of the classic sand table for military analysis. In essence, such a combat simulation is an analogue model, which recreates the sand table with the help of the digital computer, and battles are acted out on this automated sand table.

Simulation may or may not involve actual human beings playing some
of the decision-making roles in the system modelled. For the purposes of our present discussion, we will limit ourselves to so-called machine simulation that runs on a computer entirely without human participation.\textsuperscript{15} Moreover, for convenience we will henceforth refer to machine simulation simply as simulation.

Simulation is probably the most widely used technique for military systems analysis. To develop a simulation of combat operations, the military system and associated activities are microscopically studied and decomposed into a set of basic events, which in turn are ordered in sequence of occurrence (much like a network). When such a model is run to predict combat outcomes such as numbers of casualties of various types, territory lost, resources expended, etc; the battle is essentially "acted out on the computer," with the sequence and flow of events and combat activities followed in the same microscopic sequencing as determined by previous analysis. Human decision making in the combat is simulated with predetermined decision tables or rules.

Moreover, there are some problem areas that are more or less unique to the simulation of combat operations. A major problem area is the representation of terrain, especially the modelling of the line-of-sight process. A high-resolution simulation such as DYNTACS [7; 19] may spend as much as 60 percent of its running time in checking for intervisibility (i.e. the existence of line-of-sight) between weapon systems, and usually at least about 20 percent of its running time is so spent [69]. Thus, an inordinately large amount of time is usually spent in simulating the line-of-sight process in combat simulations. Terrain modelling sometimes receives attention in books on simulation (see EVANS, WALLANCE, and SUTHERLAND [26]), but usually it does not (see, for example, FISHMAN [29]). Other
problem areas (not only for simulation but for combat modeling in general) are the modeling of battlefield intelligence, route selection, and tactical decision processes (especially those relating to the management of large-scale warfare [10]).

Most combat simulations used in defense planning are so-called Monte Carlo simulations because statistical sampling techniques (involving the generation of pseudorandom numbers [29]) are used to determine the outcomes of random events, such as the outcome of firing at a target. Because of the tremendous quantity of computations and other information processing requirements in such a simulation, the use of a modern high-speed digital computer is essential. Probability distributions for all the random elements (i.e. random variables) in the simulation are required as inputs, and consequently a high-resolution Monte Carlo simulation such as DYNTACS requires a rather extensive data base for its running.\(^\text{16}\) The difficulties and costs of data base preparation are considerable and are frequently underestimated. The simulation then empirically generates the probability distribution for the set of possible combat outcomes. Each run of the simulation for a given set of input data is essentially a sample from the distribution of outcomes, and the simulation must be run repeatedly to obtain accurate statistical information about this distribution of combat outcomes.

The strong point of Monte Carlo combat simulation is that such a simulation may contain a lot of detail and therefore may be more credible than a more abstract model to many people. Examples\(^\text{17}\) of such Monte Carlo simulations are ASARS II, CARMONETTE, DYNTACS, and SIAF. Some people (see SHUBIK and BREWER [86], for example) feel, however, that such simulations make a "fetish of realism." The large amount of detail, moreover, causes a significant amount of computer time to be required for a single run of such a simulation, and this characteristic is essentially their undoing as far as being a viable analysis technique for exploring the limits

16
of system capability.

There are a number of serious shortcomings to the use of Monte Carlo simulation for defense analysis. First, such simulations are quite costly to build. It is not unreasonable to expect to spend 5 to 10 man-years of effort to develop a detailed simulation of tactical combat. Second, they are costly to run, with typically 10-20 minutes of computer time (IBM 360/67) required per replication of about the same length of battle time, and one needs 10-60 replications for statistical stability in the results (see, for example, ZIMMERMAN [120, p. 741]). Additionally, because of the amount of detail involved, the data-base requirements are quite demanding. For example, it is not unheard of to have several analysts spend about three months preparing a new set of input data and the corresponding data deck for DYNTACS. Not only is a so-called high-resolution combat simulation costly to build and run, but it is also costly to maintain: a staff of fairly highly trained personnel must be maintained to insure that the computer program stays running and debugged as changes are continually implemented. For several reasons (e.g. size of the computer program, complexity of the model, etc.), changes may be quite difficult to implement in such a combat simulation. The tremendous amount of detail (i.e. the large number of variables and other parameters) present in a simulation essentially precludes the running of parametric studies to examine the sensitivity of the model to changes in simulation assumptions and input data. Because of this lack of capability to run parametric studies, it is essentially impossible to use simulation by itself as a vehicle for determining those system capabilities, tactics, and environmental characteristics that significantly influence the system's effectiveness. As S. BONDER points out [11, Chapter 1], simulation is essentially too detailed to be by itself a useful tool for analysis. These disadvantages
of Monte Carlo simulation are summarized in Table 1.1.

Analytical models (like machine simulation) do not involve human participation during running. They may, of course, be either deterministic or stochastic in nature. Their distinguishing characteristic is their degree of abstraction: as Figure 1 shows, analytical models are more abstract than simulations. In fact, a good analytical model is usually quite abstract, poor in the number of variables explicitly considered, but rich in ease of manipulation and clarity of insight [86]. Before the advent of high-speed digital computers, an analytical model consisted of at most a few equations (see LANCHESTER's [51] classic models discussed in Chapter 2). Today large-scale processes and systems can be modelled by many equations with the help of a digital computer. The process under study is analyzed and abstracted (i.e. decomposed into basic events and activities). Then mathematical submodels of events and activities are developed and integrated into an overall structure.

Analytical models of any degree of complexity usually do not yield convenient analytical solutions but require numerical approximation methods and a digital computer for the generation of numerical results. However, in those cases in which an explicit analytical solution can be obtained, one has obviously simplified the process of understanding the model. Insights into the dynamics of combat may be obtained by, for example, examining explicit relations between the independent variables, the model's parameters, and the dependent variables (which are usually related to the MOEs). Such insights are much more difficult to acquire when the solution is not simply expressible in terms of elementary functions and, for example, finite-difference methods must be used to generate numerical (approximate) results, although the model's basic structure is explicitly contained in equations that are readily examined. Thus, although more abstract than simulations,
<table>
<thead>
<tr>
<th>Disadvantages of Monte Carlo Simulation of Combat</th>
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<tr>
<td>(D1) Costly to build</td>
</tr>
<tr>
<td>(D2) Costly to run</td>
</tr>
<tr>
<td>(D3) Costly to maintain</td>
</tr>
<tr>
<td>(D4) Lack of flexibility for change</td>
</tr>
<tr>
<td>(D5) Essentially impossible to perform sensitivity and other parametric studies</td>
</tr>
</tbody>
</table>
analytical models are characterized by their transparency (i.e. ease of revealing their basic structure and assumptions). We will focus on such models in this book.

Analytical models, particularly simple ones, help clarify the relationship between theoretical models, empiricism, and data gathering. An analytical model is usually too simple and restricted to directly solve an actual operational problem. But because of its transparency, the analytical model can warn about potential problem areas, indicate where additional measurements are most needed, and identify and order important omissions from the model (see SHUBIK and BREWER [86] for a further discussion).

There is one further general type of combat model that merits our attention, a mixture of two of the above types called the hybrid analytical-simulation model [10]. It has been developed in response to the needs for parametric analysis coupled with the long preparation and run times for Monte Carlo simulations. It combines the strengths of these two modelling approaches by representing some processes in one way and others in the other. Again, the modern high-speed digital computer makes possible the integration of these model types. For example, in battalion-level combat models such as BONDER/IUA (see [92]; also [11; 12]) (and its various derivatives such as BLDM, FAST [13], AMSWAG [36], IHA [104]) and COMAN [18], attrition and target acquisition (and sometimes allocation) processes are modelled analytically, while simulation is used to model battlefield movement processes [10]. The same general approach has been applied to large-scale combat (i.e. combat between division-size and large units) with models such as DIVOPS [106] and VECTOR-2 [107] in which the attrition, maneuver-unit-element and fire-support-sensor acquisition, and terrain-line-of-sight processes are modelled analytically [10]. Such hybrid models use LANCHESTER-type equations (i.e. deterministic differential equations) to represent the
combat attrition process.

A related (but yet distinct) classification of combat models would be according to how they assess the outcomes of tactical engagements (irrespective of how tactical decision making is modelled). Three current approaches for predicting the effectiveness of combat units in such engagements are as follows (see BONDER and FARRELL [11] for further details):

(A1) firepower scores (see STOCKFISCH [90, pp. 6-27]),
(A2) Monte Carlo simulation [33; 120],
(A3) analytical models (e.g. differential equations) [11].

All three approaches have been used to assess the outcomes of combat engagements in war games. We have already discussed Monte Carlo simulation and analytical models above so it remains to discuss the other combat-assessment approach, firepower scores. We will also say some additional words about analytical models in the context of assessing the outcomes of tactical engagements. Finally, we will briefly discuss the relation between the scale of combat operations and these modelling approaches.

The firepower-score approach is basically a technique for aggregating heterogeneous forces (i.e. tanks, artillery, infantry, etc.) into a single homogeneous force on each side. It is an index-number approach, which develops one number (referred to as the firepower index) to represent the "combat potential" of a unit. A linear model is used to develop this index number, i.e. the firepower index, from the scores of individual weapon systems as Table 1.II shows. Moreover, as emphasized by STOCKFISCH [90, p. 7], the words score and index should not be regarded as being synonymous. It is more precise, therefore, to use the term firepower score to refer to the military capability or value of a specific weapon system and to use the term firepower index -- which is obtained by summing scores --
### TABLE 1.II. Hypothetical Example of Determination of Firepower Index for a Combat Unit

<table>
<thead>
<tr>
<th>Weapon Type</th>
<th>Number</th>
<th>Firepower Score</th>
<th>Total Contribution to Firepower Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rifle, M-16, 5.56mm</td>
<td>6,000</td>
<td>1</td>
<td>6,000</td>
</tr>
<tr>
<td>MG, M-60, .30 cal</td>
<td>150</td>
<td>6</td>
<td>900</td>
</tr>
<tr>
<td>MG, M-2, .50 cal</td>
<td>250</td>
<td>10</td>
<td>2,500</td>
</tr>
<tr>
<td>Mortar, M-125, 81mm</td>
<td>50</td>
<td>20</td>
<td>1,000</td>
</tr>
<tr>
<td>Howitzer, M-109(SP), 155mm</td>
<td>50</td>
<td>40</td>
<td>2,000</td>
</tr>
<tr>
<td>Howitzer, 8&quot;</td>
<td>8</td>
<td>30</td>
<td>240</td>
</tr>
<tr>
<td>Tank, M60A2</td>
<td>200</td>
<td>100</td>
<td>20,000</td>
</tr>
</tbody>
</table>

**TOTAL FIREPOWER INDEX 32,640**

Firepower Index for U.S. Army's 7th Infantry Division
to refer to the military capability or value of some aggregation of diverse weapons. In other words, the firepower-score approach provides a common denominator for aggregating the many different types of weapons on a battlefield, and military combat is characterized by such "combined-arms" operations consisting of many different weapon systems.

How is the basic firepower score for a weapon system determined? There are apparently almost as many different answers to this crucial question as there are different firepower-score methods. Many methods state that the firepower score of a weapon system is essentially the product of a measure of single-round lethality multiplied by the expected expenditure of ammunition during a fixed period of time. Although this procedure appears to yield an objective measure of weapon-system capability, STOCKFISCH [90, pp. 23-78, especially pp. 23-27 and 76-78] points out that actually varying amounts of subjectivity are cranked into various such firepower scores. Moreover, the firepower-score approach probably dates back to World War II, although documentation about it is generally somewhat difficult to come by (see STOCKFISCH [9] for introduction to the scanty firepower-score literature).

In large-scale (i.e. division-level and above) ground-combat models, firepower indices are used as a surrogate for unit strength. They are then in general used to:

(U1) determine engagement outcomes,
(U2) assess casualties,
(U3) determine FEBA movement.

[FEBA stands for Forward Edge of the Battle Area. It is the contact zone between two opposing forces.] The force ratio is the significant factor in such determinations. Here the term force ratio means the ratio of the firepower index (i.e. the aggregation of all the firepower scores in the unit)
of the attacker to that of the defender. Let us consider a hypothetical example to illustrate this point. Consider, for example, the 7th Division of the U.S. Army and assume that the firepower scores shown in Table 1.2 apply. Then the 7th Division has a firepower index of 32,640. If an attacking enemy Army Group were to have a firepower index of 146,880, then we would have a force ratio of 4.50 (A/D), where A refers to the attacker and D to the defender.

Although the firepower-score approach has been widely used for top-level planning, it has received increasing criticism in recent years (see, for example, STOCKFISCH [90] or [11]). Significant deficiencies of the index-number approach are the following (from [11]):

(D1) it does not measure the accomplishment of unit missions,

(D2) it ignores most of the significant factors that affect mission accomplishment (i.e. weapon system characteristics, threat variables, organizational structures, tactics employed, environmental conditions, etc.),

(D3) it oftentimes bears little relation to the physical combat or other processes under study.

STOCKFISCH [90, p. 128] claims that no satisfactory simple technique for aggregating modern conventional forces currently exists. Although the firepower-score approach has been thus far much criticized, conventional forces must be aggregated in many analyses, and until a better alternative is developed, firepower scores will continue to be used.

Analytical models have been discussed in general terms above. We will now discuss their use specifically for assessing the outcomes of combat engagements. In particular, differential-equation models have been fairly widely used for the assessment of combat outcomes. Such models are
used to represent the decay in numbers of weapon systems (i.e. the attrition process) and require submodels (again usually analytical ones) for various subordinate processes such as target detection, target location, fire allocation, etc. The modern large-scale digital computer has made possible the development of large-scale hierarchical system models, with submodels feeding information into a master coordinating model. In the field of combat modelling, the basic calculation is one of force attrition, and consequently is usually done with the aid of some type of differential-equation model. The use of such models as practical analysis tools is primarily due to the efforts of S. BONDER and his colleagues formerly at the University of Michigan and now at Vector Research, Inc. Their main contribution has been the development of fairly detailed submodels for the prediction of loss rates from engineering and operational data for such differential-equation models. We will refer to such a differential-equation model that represents attrition from enemy action through a system of differential equations for the force levels as a *LANCHESTER-type model of warfare* (also commonly called a *differential combat model* [16]). The rest of this book concerns such models.

Each of the above combat-assessment approaches (i.e. firepower scores, Monte Carlo simulation, and analytical models) may be thought of as corresponding to a different scale of combat operations, with the firepower-score approach and Monte Carlo simulation being at opposite ends of the spectrum of the scale of combat operations (i.e. the size of the units involved). This correspondence is shown in Table 1.III. The contents of Table 1.III are only generally true, with exceptions certainly existing. As we see from this table, the firepower-score approach has been primarily used for engagement assessments in large-scale (i.e. theater-level) combat
<table>
<thead>
<tr>
<th>Modelling Approach</th>
<th>Scale of Combat of Example Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>firepower score</td>
<td>theater - ATLAS, CEM</td>
</tr>
<tr>
<td>Monte Carlo simulation</td>
<td>infantry: platoon - ASARS II</td>
</tr>
<tr>
<td></td>
<td>armor: company/battalion -</td>
</tr>
<tr>
<td></td>
<td>DYNTPAC, CARMONETTE</td>
</tr>
<tr>
<td>LANCHESTER-type model</td>
<td>battalion - BONDER/IUA</td>
</tr>
<tr>
<td></td>
<td>division - DIVOPS</td>
</tr>
<tr>
<td></td>
<td>theater - VECTOR-2, TWSP,</td>
</tr>
<tr>
<td></td>
<td>BALFRAM, DMEW</td>
</tr>
</tbody>
</table>

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models. Although there are exceptions, high-resolution Monte-Carlo simulation has been a feasible assessment approach only when there have been no more than about 100 elements (e.g. individual tanks, crew-served weapons, etc.) on each side. On the other hand, LANCHESTER-type models have been developed for the full spectrum of combat operations, from combat between company/battalion-sized units to theater-level combat operations.

1.4. The Influence of Modern-Digital Computer Technology. 24

Without the modern high-speed digital computer both high-resolution Monte Carlo simulations such as DYNAMICS and CARMONETTE and also differential combat models such as BONDER/IUA and its many derivatives would be impossible. The modern computer provides not only large-scale memory capacity but also the ability to perform millions of arithmetic operations per second. In such a computational environment, the numerical integration of a system of hundreds of ordinary differential equations becomes possible. Today LANCHESTER-type complex system models, which rely on modern digital computer technology for their implementation (see, for example, BONDER and HONIG [12]), have been developed for various levels of combat, from combat between battalion-sized units (see BOSTWICK et al. [13] or HAWKINS [36]) to theater-level operations (see CORDESMAN [21], FARRELL [28], or [105; 107]).

1.5. The Purpose of This Book.

As indicated above, there currently appears to be a trend toward increasing interest in LANCHESTER-type models of warfare. However, information about the nature of such models, their strengths and weaknesses, etc., unfortunately does not appear to be widely disseminated beyond a relatively small group of research workers. Moreover, there have been essentially no readily

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accessible sources of general information about LANCHESTER-type models: there has been no book, textbook, or monograph on LANCHESTER-type models of warfare, and the one and only survey article by DOLANSKY [23] appeared in 1964. Considering contemporary developments, DOLANSKY's article is quite out of date today. Furthermore, results and developments have been widely scattered in the literature, and it has been difficult (if not impossible) for an analyst to obtain general information and an overview of LANCHESTER-type models of warfare.

The purpose of this book is to provide a comprehensive survey of LANCHESTER-type models of warfare. By LANCHESTER-type models of warfare we mean differential-equation models that describe changes over time in the force levels of the combatants and other significant variables that describe the combat process. Our objective is to present a unified treatment of such models and of their behavior, with emphasis on the insights that may be consequently obtained into the dynamics of combat. We hope to tie together much of the knowledge about LANCHESTER-type models that has been heretofore widely scattered in the literature.

In the past (say up until about 1970), LANCHESTER-type models of warfare were only used by a small group of the leading analysts: as a consequence of pioneering work by F. W. LANCHESTER [51] done about the time of World War I, a few military operations analysts have used simplified deterministic differential-equation models to develop insights into the dynamics of combat from about the end of World War II (see, for example, [8; 11; 12; 23; 94; 110-112]). The advent of the modern high-speed digital computer has made feasible the development and use of quite complicated versions of such LANCHESTER-type (also frequently called differential) models as practical defense planning tools [10]. Thus, today militarily
realistic computer-based LANCHESTER-type models of quite complex combat systems have been developed and are fairly widely used by a much larger number of analysts than ever used the simple differential-equation models. Thus, the modern digital computer has made much more extensive use of these models possible. Such models currently exist for almost the entire spectrum of combat operations, from combat between battalion-sized [13] and division-sized [16] units to theater-level operations [21; 28]. The study of the basic nature and behavior of such differential combat models is the subject of this book. Our goal is to promulgate a better understanding of such models.

Two divergent aspects of LANCHESTER-type combat models are the following:

(A1) insights that they provide into the dynamics of combat,

(A2) their enrichment in order to better model real-world combat activities.

As is always the case, a book reflects the tastes and interest of its author. Inspired by the works of F. W. LANCHESTER and H. K. WEISS, I have been more interested in obtaining insights into the dynamics of combat from relatively simple models than enriching such models in details (see W. T. MORRIS [63] for a discussion of the processes of such enrichment). Hence, this book emphasizes studying relatively simple combat models in order to learn their basic nature and to, hopefully, perceive significant interrelationships that are difficult to discern in more complex models. Such insights can provide valuable guidance for more detailed computerized investigations (see WEISS [112]). We will also consider the use of LANCHESTER-type models of warfare for developing quantitative insights into optimal time-sequential combat strategies (see Chapter 8).
1.6. **Dynamic Systems and State Variables.**

The LANCHESTER-type combat models considered in this book may be viewed from the vantage point of system theory (see PADULO and ARBIB [68]). We will find it convenient to do so in order to better understand the philosophical underpinnings of such models. Let us therefore introduce the reader to some intuitive notions and ideas related to systems. We will not attempt to give explicit and precise definitions. For our purposes intuitive and rather vague terminology will suffice.  

A **physical system is defined as an interconnection of physical elements, or objects.** The notion of a system is rather broad: it applies not only to simple mechanical and electrical devices but also to more esoteric and complex systems such as automobiles and (especially) weapons systems. In particular, one can view military units such as companies and battalions as systems.

Systems may be either static or dynamic. This book concerns dynamic combat systems. For our purposes, a **dynamic system is one whose inputs and outputs are related by a set of differential (or difference) equations.** The system evolves dynamically over time. The set of differential equations provides a model for the system's evolution. We require that such a model be valid in the sense that the present predicts the future. Let us informally, therefore, introduce the notion of cause and effect or, more formally, the **principle of casualty.** Consider the following example: in **NEWTONIAN mechanics, the future motion of a system of particles is completely determined if the present positions and moments are known, along with the present and future forces.** Future forces have no affect on the present (nonanticipatory system), and how the system reached its present state is not important.
Knowledge of the present allows us to predict the future. What we must know about the present (besides the equations that describe the evolution of such quantities) is called the state of the system. Intuitively, the state of a system is the minimum amount of present information about the history of the system that allows one to predict the effect of the past upon the future. The variables that are used to describe the state of a system are called the state variables.

The above terminology is convenient for communication about LANCHESTER-type models of warfare. Later when we consider time-sequential combat strategies, it will be convenient to introduce the system-theory notions of closed-loop and open-loop controls. As we will see in the next chapter, one may view LANCHESTER's classic combat theory as saying that force levels are the state variables for combat between two military systems. We return to this theme later.

1.7. Final Remarks.

Thus, we see that we may say that LANCHESTER-type models of warfare represent dynamic combat systems whose state variables are typically force levels. In this introductory chapter we have established a framework for studying such differential-equation models of combat: we have examined the general nature of models, the use of combat models in defense planning in the United States, and the various types of combat models that are in current use. Based on our examination of the scientific study of conflict and warfare, we feel that most of the shortcomings usually attributed to LANCHESTER-type models are also the shortcomings of any combat model.

Moreover, we feel that LANCHESTER-type models are an ideal vehicle for studying combat dynamics because of the relative ease of extracting information from them and the fact that usually no other type of model is better justified.
Our conclusion is based on a careful examination of the state-of-the-art of conflict and combat modelling. In the next chapter we will see how LANCHESTER-type models readily provide many important insights into the dynamics of combat.